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Analytic Study of Disoriented Chiral Condensates *

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Abstract

By introducing a quark source in the nonlinear σ model, we obtain an analytic boost-invariant solution as a candidate for the disoriented chiral condensate (DCC) in 3+1 dimensions. In order to trigger formation of the DCC, a strong transfer of axial isospin charge must occur between the expanding source and the interior in the baked Alaska scenario. An explicit chiral symmetry breaking is incorporated in the isospin-uniform solution by connecting the decay period to the formation period. Quantitative estimates are presented with our simple solution. At least in this class of solutions, the explicit symmetry breaking masks almost completely the disorientation which would be reached asymptotically in the symmetric limit.

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Disclaimer

1 Introduction

It has been speculated that disoriented chiral condensates (DCCs) may be produced in high-energy hadron collisions and heavy-ion collisions [1]. In Bjorken's baked Alaska scenario [2], as a hot dense matter spreads from a collision center, a disoriented vacuum is created in its hollow interior. The disoriented vacuum eventually relaxes to the true vacuum by emitting the excess energy in soft pions. Many numerical calculations have been performed with the σ model to study if a DCC can really be formed [3]. Though they often find a long range correlation leading to a DCC-like state, it is not long enough to lead to a spectacular Centauro or anti-Centauro event [4]. Search for analytic solutions has also been done in the σ model [5, 6, 7]. It shows among others that formation of isospin aligned DCCs is strongly suppressed by the phase space of chiral rotations [7].

Formation and decay of the DCC can be treated separately. During the formation period, we may ignore the explicit chiral symmetry breaking due to the u and d current quark masses. As the pion field cools down and its kinetic energy becomes comparable with the energy scale of the explicit symmetry breaking, the approximation of chiral symmetry breaks down and thereafter the symmetry breaking plays a major role. The symmetry breaking causes attenuation of the classical pion field, and eventually quantum fluctuations dominate over the classical field. In other words, the classical field decays away by emitting pion quanta.

In this paper we study analytically the formation and decay of the DCC by the nonlinear σ model at zero temperature. We assume that there exists a window of time period, sometime after the initial stage of collision but before the beginning of decay, where the *chiral symmetric* classical σ model is a good approximation. Our main purpose is to learn what initial condition triggers a DCC formation and how the DCC evolves subsequently. We do not ask how a desired initial condition is created. We will make one conceptual departure from our previous viewpoint [7] with regard to what we call the DCC in the σ model. The DCC would not decay if there were not for an explicit chiral symmetry breaking. For this reason, the terminal state of the pion field obtained in the chiral *symmetric* σ model should be identified with the initial state of the decaying DCC. Therefore the DCC pion field should approach a nonvanishing

asymptotic one at $t \to \infty$ if a symmetry breaking is turned off.

In Sec.II we formulate our approach with the nonlinear σ model coupled to quarks. In Sec.III we consider several quark sources that are of particular interest to the DCC formation in $1+1$ and $3+1$ dimensions. While a boost-invariant solution never reaches a static asymptotic configuration in $1+1$ dimensions, we find in $3+1$ dimensions a simple interesting source term leading to a boost-invariant pion field which approaches the true vacuum at $t \to \infty$. In Sec.IV, by making $SU(2) \times SU(2)$ rotations on this $3+1$ solution, we obtain the solutions in which the pion field approaches an isospin-uniform static configuration everywhere off the light cone. These solutions should be interpreted as the DCC. In Sec.V, we introduce an explicit symmetry breaking and obtain a complete spacetime evolution of the simple isospin-uniform solution of Sec.IV by smoothly continuing the formation period to the decay period. Because the ratio of the symmetry breaking quantity m_π to the symmetric quantity f_π is not small numerically, the transition from the formation period to the decay period is obscure for most DCC solutions. In Sec.VI, we interpret our findings from the viewpoint of conservation of the axial isospin charge. It will help us to understand what can possibly lead to creation of the favorable quark sources and what is the chance to realize a favorable initial condition.

2 Source of pion field

In the environment of DCC formation, the matter particles are presumably in the quark-gluon phase rather than in the nucleon phase. The appropriate low-energy effective Lagrangian is therefore the σ model coupled to quarks and antiquarks. In terms of the chiral or current quark $q_{R,L}$ of u and d, the Lagrangian in the $SU(2) \times SU(2)$ symmetry limit is:

$$\begin{aligned} \mathcal{L} &= \frac{f_\pi^2}{4}\mathrm{tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + i\overline{q}_R \not{\partial} q_R + i\overline{q}_L \not{\partial} q_L \\ &\quad - g f_\pi \overline{q}_L \Sigma q_R - g f_\pi \overline{q}_R \Sigma^\dagger q_L, \end{aligned} \tag{1}$$

where

$$\Sigma = e^{i\boldsymbol{\tau}\cdot\mathbf{n}(x)\theta(x)}. \tag{2}$$

Three isospin components of pion field are identified with

$$\boldsymbol{\pi}(x) = f_\pi \mathbf{n}(x)\,\theta(x), \tag{3}$$

where $\mathbf{n}(x)^2 = 1$. We choose the nonlinear representation for the pion field instead of the linear representation since we can impose more easily the condition that the $\pi - \sigma$ fields be near the bottom of potential well. By fixing the radial σ field to f_π, we narrow the region of applicability of the σ model to the energy range where the pion kinetic energy is much smaller than the depth of the *Mexican hat* potential:

$$\frac{1}{2}\dot{\boldsymbol{\pi}}^2 \ll \frac{1}{8} m_\sigma^2 f_\pi^2. \tag{4}$$

We move from the current quark $q_{R,L}$ to the constituent quark $Q_{R,L}$ by

$$Q_R = \xi\, q_R, \quad Q_L = \xi^\dagger q_L, \tag{5}$$

with

$$\xi = e^{i\boldsymbol{\tau}\cdot\mathbf{n}(x)\theta(x)/2}. \tag{6}$$

Use of the constituent quark does not mean that the quarks are static in the expanding shell. It is because the equation of motion takes a simpler form. The Lagrangian is now expressed in $\theta(x)$, $\mathbf{n}(x)$, and $Q_{R,L}(x)$ as

$$\begin{aligned} \mathcal{L} \;=\; & \frac{f_\pi^2}{2}(\partial_\mu\theta\partial^\mu\theta + \sin^2\theta\,\partial_\mu\mathbf{n}\cdot\partial^\mu\mathbf{n}) + \frac{\lambda f_\pi^2}{2}(\mathbf{n}^2 - 1) \\ & + i\overline{Q}_R\,\partial\!\!\!/ Q_R + i\overline{Q}_L\,\partial\!\!\!/ Q_L - m_Q(\overline{Q}_L Q_R + \overline{Q}_R Q_L) \\ & + i\overline{Q}_R\xi\,\partial\!\!\!/\xi^\dagger Q_R + i\overline{Q}_L\xi^\dagger\,\partial\!\!\!/\xi Q_L, \end{aligned} \tag{7}$$

where m_Q is the constituent quark mass given by $m_Q = g f_\pi$, and λ is a Lagrange multiplier.

The Euler-Lagrange equation for θ is:

$$\Box\theta - \sin\theta\,\cos\theta\,\partial_\mu\mathbf{n}\cdot\partial^\mu\mathbf{n} = -i\frac{m_Q}{f_\pi^2}\mathbf{n}\cdot(\overline{Q}\boldsymbol{\tau}\gamma_5 Q). \tag{8}$$

After λ is eliminated, the Euler-Lagrange equation for $\mathbf{n}$ becomes

$$\partial_\mu(\sin^2\theta\,\mathbf{n}\times\partial^\mu\mathbf{n}) = -i\frac{m_Q}{f_\pi^2}\mathbf{n}\times(\overline{Q}\boldsymbol{\tau}\gamma_5 Q)\sin\theta. \tag{9}$$

We treat the quark field as a given external source.

Through global $SU(2) \times SU(2)$ rotations a solution with a uniform isospin orientation ($\mathbf{n}(x) = constant$) generates infinitely many solutions with $\mathbf{n}(x) \neq constant$ that are degenerate in energy [6, 7]. It has been pointed out that in the boost-invariant $1+1$ dimensional case, the chiral rotations of isospin-uniform solutions generate all known solutions [7]. A wide class of isospin-nonuniform solutions can be obtained in this way in $3+1$ dimensions too. It is much easier to make chiral rotations of a uniform solution than to solve directly for general nonuniform solutions. We study here the class of solutions that can be rotated into a uniform one by chiral rotations, *i.e.*, *Anselm-class* solutions according to the nomenclature of Bjorken. The solutions of constant $\mathbf{n}$ are realized when the source points to a certain fixed direction everywhere:

$$-i\Big(\frac{m_Q}{f_\pi^2}\Big)\overline{Q}\boldsymbol{\tau}\gamma_5 Q = \mathbf{n}_0\rho(x), \tag{10}$$

where $\rho(x)$ is a real Lorentz pseudoscalar function. Once we set the isospin direction as in Eq.(10), the source term in the equation of motion for $\mathbf{n}$ disappears if we choose

$$\mathbf{n}(x) = \mathbf{n}_0. \tag{11}$$

Thus a configuration of uniform isospin orientation satisfies the equation of motion for $\mathbf{n}(x)$ trivially. The equation of motion for $\theta(x)$ then turns into

$$\Box\theta = \rho(x). \tag{12}$$

This is the equation that we will focus on in Sec.III. Since we have already fixed the radial σ field to its expectation value f_π, the scalar quark density $\overline{Q}Q$ does not contribute to formation of the pion field. It is the isovector pseudoscalar density of the *constituent* u and d quarks that excites the phase pion field $\mathbf{n}_0\theta(x)$.

3 Quark densities of physical interest

We study the pion field generated by the typical quark sources which are particularly interesting to the DCC formation. We start with the case of $1+1$ dimensions, which is viewed as the $\mathbf{p}_t = 0$ limit of the real world.

A. $1+1$ **dimensions**

In order to solve Eq.(12), we need the Green function of $\Box\theta = 0$. The retarded Green function is

$$G(x;x') = \frac{1}{4}\Theta(t-t')\Big(\epsilon(t-t'-|z-z'|)+\epsilon(t-t'+|z-z'|)\Big), \tag{13}$$

where $\Theta(t)$ is the step function and $\epsilon(t) = \Theta(t) - \Theta(-t)$. The simplest source is a pair of boost-invariant densities flying away to opposite directions in the speed of light. This source is of practical interest since a boost-invariant source in the coordinate space gives a boost-invariant pseudo-rapidity particle distribution. With the variable $\tau' = \sqrt{t'^2 - z'^2}$, we express the source $\boldsymbol{\rho}(x') = -i(m_Q/f_\pi^2)(\overline{Q}\boldsymbol{\tau}\gamma_5 Q)$ as

$$\boldsymbol{\rho}(x') = \mathbf{q}_0\delta(\tau'^2)\Theta(t'), \tag{14}$$

where $\mathbf{q}_0$ is a constant vector in isospin space. This form is the most general one that satisfies boost invariance, barring derivatives of $\delta(\tau'^2)$. The source has a common isospin orientation at the both sides ($z' = t'$ and $z' = -t'$). It is singular at the origin $z' = t' = 0$ and dies away like $1/|z'|$ at far distances. Since the initial state at $t' = z' = 0$ is outside the region of applicability of the σ model, we will regularize $\boldsymbol{\rho}(x')$:

$$\mathbf{n}_0\theta(x) = \int_\varepsilon^t dt'\Big(\int_{-\infty}^{-\varepsilon} dz' + \int_\varepsilon^\infty dz'\Big)G(x;x')\boldsymbol{\rho}(x'), \tag{15}$$

where x and x' denote (t,z) and (t',z'), respectively. Since the integral over z' actually diverges if it is extended to time $t' = 0 = z'$, a small neighborhood of $|z'| \leq \varepsilon$ has been excluded in the z' integral. The integrals over z' at $\varepsilon < z' < \infty$ and $-\infty < z' < -\varepsilon$ are not separately boost invariant, but the sum is. The result is

$$\mathbf{n}_0\theta(x) = \frac{1}{2}\mathbf{q}_0 \ln\left(\frac{\tau}{2\varepsilon}\right), \tag{16}$$

where $\tau = \sqrt{t^2 - z^2}$. This logarithmic solution has been well known [5, 7]. It is peculiarity of $1+1$ dimensions that the boost-invariant pion field does not die away as $t \to \infty$ even at locations off the light cone. By global $SU(2) \times SU(2)$ rotations of this solution, we can generate infinitely many more solutions, which are also known.

It may be interesting to study for comparison the case opposite to boost invariance by flipping the isospin direction for one side of the source pair, say,

at $z' = -t'$. If the source splits statistically at random at $t = 0$ the opposite isospin orientation will occur more frequently than the parallel orientation by isospin conservation. The solution for the opposite isospin orientation is not boost invariant:

$$\mathbf{n}_0\theta(x) = \frac{1}{4}\mathbf{q}_0 \ln\left(\frac{t+z}{t-z}\right). \tag{17}$$

B. $3+1$ **dimensions**

The retarded Green function in $3+1$ dimensions is the well-known Liénard-Wiechert potential:

$$G(x; x') = \frac{1}{2\pi}\delta\Big((t-t')^2 - (\mathbf{r}-\mathbf{r}')^2\Big)\Theta(t-t'). \tag{18}$$

The isovector source that flies away in a spherical shell in the speed of light can be expressed generally in the form

$$\boldsymbol{\rho}(x') = \mathbf{n}_0\sigma(\mathbf{r}')\delta(\tau'^2)\Theta(t'), \tag{19}$$

where $\tau'^2 = t'^2 - r'^2$. We first argue what r'-dependence is physically interesting for $\sigma(\mathbf{r}')$.

It appears natural that the total integrated strength of source dies away with time or distance as it feeds the pion field. We will present an argument in favor of this behavior in the final Section. Choice of $\sigma(\mathbf{r}') = \mathit{constant}$ would make the source invariant under Lorentz boost along all directions. For such a source, however, the total source integrated over the shell would increase with time as $\sim t'$. It appears that more interesting possibility is

$$\sigma(\mathbf{r}') = \frac{\bar{\sigma}(\theta', \phi')}{r'^2}. \tag{20}$$

For this $\sigma(\mathbf{r}')$, the integrated source strength weakens like $1/t'$ or $1/r'$ since one power of $1/r'$ comes from $\delta(\tau'^2)$ in Eq.(19). If the forward and backward patches of source within fixed solid angles dominate in the formation of pion field, the source behaves exactly like the boost-invariant $1+1$ dimensional source discussed above. For this source we obtain a remarkably simple pion field,

$$\mathbf{n}_0\theta(x) = \mathbf{n}_0\frac{\int \bar{\sigma}(\theta', \phi')d\Omega'}{4\pi(t^2 - r^2)}. \tag{21}$$

The pion field is determined only by the total source integrated over the shell, independent of its distribution. Furthermore, it is boost invariant. This extremely simple result is special to the $1/r'^2$ dependence of $\sigma(\mathbf{r}')$.

If instead we demand that the source on a surface element within a solid angle $\Delta\Omega$ should remain constant of time, the r'-dependence of the source is of the form

$$\sigma(\mathbf{r}') = \frac{\tilde{\sigma}(\theta', \phi')}{r'}. \tag{22}$$

In this case the produced pion field depends not only on the total strength of source but also on the distribution on the shell. If $\tilde{\sigma}(\theta', \phi')$ is constant $(= \tilde{\sigma}_0)$, the pion field is

$$\mathbf{n}_0\theta(x) = \frac{1}{4r}\mathbf{n}_0\tilde{\sigma}_0 \ln\left(\frac{t+r}{t-r}\right). \tag{23}$$

If the isospin direction of source is opposite in sign in the forward and backward hemispheres ($\tilde{\sigma}_1 =$ constant) as

$$\tilde{\sigma}(\theta', \phi') = \tilde{\sigma}_1 cos\theta', \tag{24}$$

the produced pion field is given by

$$\mathbf{n}_0\theta(x) = \frac{1}{4t}\mathbf{n}_0\tilde{\sigma}_1\left(\frac{t^2}{r^2}\ln\frac{t+r}{t-r} - \frac{2t}{r}\right)\cos\theta \tag{25}$$

We have considered here the class of $\sigma(\mathbf{r}')$ that factorizes into r' and (θ', ϕ'). For the sources of this kind, the relative isospin strength on different parts of the source shell does not change with time. Since $t' = r'$ on the source, we can incorporate time variation of the relative isospin strength by adding nonleading $1/r'$ terms. However, it is the leading $1/r'$ term of $\sigma(\mathbf{r}')$ that determines the asymptotic pion field at $t \to \infty$. As for isospin orientation, an aligned source $i\overline{Q}\boldsymbol{\tau}\gamma_5 Q$ transforms nonlinearly under chiral rotations. Since the isospin direction is entangled with spacetime dependence in this transformation, isospin alignment of source is destroyed upon chiral rotations and the source becomes locally nonuniform. Instead of rotating the source, we can accomplish the same by rotating $\mathbf{n}_0\theta(x)$. The requirement that the total integrated pseudoscalar charge $\int \boldsymbol{\rho}(x')d^3\mathbf{r}'$ should not increase with time sets an upper bound on the r' dependence of $\sigma(x')$:

$$|\sigma(\mathbf{r}')| \leq \frac{|\sigma(\theta', \phi')|}{\mathrm{r}'} \quad (\mathrm{r}' \to \infty). \tag{26}$$

Then the asymptotic pion field at any location inside the light cone must vanish with this restriction:

$$|\theta(x)| \leq \left| \frac{1}{4\pi} \int \frac{\sigma(\theta', \phi')}{t - r\cos\psi} d\Omega' \right| \to 0 \quad (t \to \infty), \tag{27}$$

where $\cos\psi = (\mathbf{r} \cdot \mathbf{r}')/rr'$. We can see it explicitly in the sample solutions given above.

Do these isospin-uniform solutions describe DCCs in $3+1$ dimensions ? One viewpoint is that any classical pion field should be called the DCC even if it is transient. In this case the disorientation does not last forever even in the symmetric limit. We can take a different viewpoint: The DCC is a disoriented state which would persist forever if the chiral symmetry were turned off. We have so far solved the σ model in the chiral symmetry limit. Therefore, if we take this second viewpoint, what we should identify with DCCs is not the solutions obtained above, but those solutions that approach a nonvanishing limit, $\theta(\infty, \mathbf{r}) \neq 0$. Our isospin-uniform solutions are driven to the true vacuum by chiral symmetric force alone. It turns out that the issue is largely semantic rather than physical. Later when we connect these solutions to the decay solutions by including an explicit symmetry breaking, we will find that the solutions with $\theta(\infty, \mathbf{r}) \neq 0$ oscillate with a slightly larger amplitude during the decay period than those with $\theta(\infty, \mathbf{r}) = 0$. Otherwise there is little difference between them. Though difference is very minor, there is an important conceptual distinction between the solutions with $\theta(\infty, \mathbf{r}) = 0$ and those with $\theta(\infty, \mathbf{r}) \neq 0$. In this paper we call the latter as the DCC. In the next Section we will obtain the solutions with $\theta(\infty, \mathbf{r}) \neq 0$ by chiral rotations of the solutions with $\theta(\infty, \mathbf{r}) = 0$.

4 DCC solutions

In this Section we obtain the DCC solutions with $\theta(\infty, \mathbf{r}) \neq 0$ from the simple solution of Eq.(21) by chiral rotations. Hereafter we denote the solution $\mathbf{n}_0\theta(x)$ of Eq.(21) by $\mathbf{n}_0\theta_0(x)$,

$$\theta_0(x) = \frac{a}{t^2 - r^2} \quad (\to 0 \text{ as } t \to \infty), \tag{28}$$

and the Σ field of $\mathbf{n}_0\theta_0(x)$ by $\Sigma_0(x)$. Let us designate global $SU(2)_R \times SU(2)_L$ rotations by a pair of vector rotation angles, $(2\mathbf{n}_R\theta_R, 2\mathbf{n}_L\theta_L)$. They transform

the nonlinear pion field $\Sigma_0(x)$ as

$$\Sigma_0(x) \rightarrow U(\mathbf{n}_L\theta_L)\Sigma_0(x)\, U^\dagger(\mathbf{n}_R\theta_R), \tag{29}$$

where

$$U(\mathbf{n}\theta) = e^{i\boldsymbol{\tau}\cdot\mathbf{n}\theta}. \tag{30}$$

It is straightforward to find the transformation formulas for $\theta(x)$ and $\mathbf{n}(x)$ [8]:

$$\begin{aligned}\cos\theta \;&=\; \Big(c_L c_R + (\mathbf{n}_L\cdot\mathbf{n}_R)s_L s_R\Big)c_0 + \\ &\quad \Big((\mathbf{n}_0\cdot\mathbf{n}_R)c_L s_R - (\mathbf{n}_0\cdot\mathbf{n}_L)s_L c_R + (\mathbf{n}_0\times\mathbf{n}_L)\cdot\mathbf{n}_R\, s_L s_R\Big)s_0,\end{aligned} \tag{31}$$

$$\begin{aligned}\mathbf{n}\sin\theta \;&=\; \Big(\mathbf{n}_L s_L c_R - \mathbf{n}_R c_L s_R + (\mathbf{n}_L\times\mathbf{n}_R)s_L s_R\Big)c_0 \\ &\quad + \Big(\mathbf{n}_0 c_L c_R + (\mathbf{n}_0\times\mathbf{n}_L)s_L c_R + (\mathbf{n}_0\times\mathbf{n}_R)c_L s_R \\ &\quad + ((\mathbf{n}_0\cdot\mathbf{n}_L)\mathbf{n}_R + (\mathbf{n}_0\cdot\mathbf{n}_R)\mathbf{n}_L - (\mathbf{n}_L\cdot\mathbf{n}_R)\mathbf{n}_0)s_L s_R\Big)s_0,\end{aligned} \tag{32}$$

where $c_{L,R}$ and $s_{L,R}$ stand for $\cos\theta_{L,R}$ and $\sin\theta_{L,R}$, respectively. Since $\Sigma_0(x)$ approaches the unit matrix at $t=\infty$, the asymptotic pion field of the rotated solution is given by

$$\Sigma(\infty,\mathbf{r}) = U(\mathbf{n}_L\theta_L)U^\dagger(\mathbf{n}_R\theta_R). \tag{33}$$

In terms of the θ and $\mathbf{n}$ fields,

$$\cos\theta(\infty,\mathbf{r}) = \cos\theta_L\cos\theta_R + (\mathbf{n}_L\cdot\mathbf{n}_R)\sin\theta_L\sin\theta_R, \tag{34}$$

$$\begin{aligned}\mathbf{n}(\infty,\mathbf{r})\sin\theta(\infty,\mathbf{r}) \;&=\; \mathbf{n}_L\sin\theta_L\cos\theta_R \\ &\quad - \mathbf{n}_R\cos\theta_L\sin\theta_R + (\mathbf{n}_L\times\mathbf{n}_R)\sin\theta_L\sin\theta_R.\end{aligned} \tag{35}$$

If we choose as a special case the rotations of $\mathbf{n}_R = \mathbf{n}_L$, the asymptotic $\theta - \mathbf{n}$ fields take a very simple form:

$$\theta(\infty,\mathbf{r}) = \theta_L - \theta_R, \quad \mathbf{n}(\infty,\mathbf{r}) = \mathbf{n}_L. \tag{36}$$

In this rotated solution, the pion field points asymptotically to the direction of $\mathbf{n}_L(=\mathbf{n}_R)$ with magnitude $\theta_L - \theta_R$ everywhere inside the light cone. At finite time, according to Eq.(32), the rotated $\mathbf{n}(x)$ field is nonuniform in isospin

direction. If furthermore $\mathbf{n}_L$ and $\mathbf{n}_R$ are chosen along the directions of $\mathbf{n}_0$ of the solution $\mathbf{n}_0\theta_0(x)$, the rotated solution turns into an almost trivial form:

$$\mathbf{n}\theta(x) = \mathbf{n}_0\Big(\theta_L - \theta_R + \theta_0(x)\Big). \tag{37}$$

This relation is a direct consequence of $SU(2)_R \times SU(2)_L$ symmetry of Lagrangian. When $\mathbf{n}_L$ $(= \mathbf{n}_R)$ does not coincide with $\mathbf{n}_0$, we can still find relatively simple formulas that describe the asymptotic behavior of the nonuniform solution:

$$\theta(x) \to \theta_L - \theta_R + (\mathbf{n}_L \cdot \mathbf{n}_0)\theta_0(x) + O(\theta_0(x)^2), \tag{38}$$

$$\begin{aligned} \mathbf{n}(x) \quad \to \quad & \mathbf{n}_L + \frac{\sin(\theta_L + \theta_R)}{\sin(\theta_L - \theta_R)}(\mathbf{n}_0 \times \mathbf{n}_L)\theta_0(x) \\ & + \frac{\cos(\theta_L + \theta_R)}{\sin(\theta_L - \theta_R)}\Big(\mathbf{n}_L \times (\mathbf{n}_0 \times \mathbf{n}_L)\Big)\theta_0(x) + O(\theta_0(x)^2), \end{aligned} \tag{39}$$

where $\theta_0(x)$ is the asymptotic tail of the uniform solution These formulas are valid for $\theta_0(x) \ll |\theta_L - \theta_R|$.

In the hypothetical world of perfect chiral symmetry, the disoriented region of a DCC solution would expand without limit and no pions would be emitted. The massless pions would sit at rest forever since they cost no energy. Actually the word "disoriented" is inappropriate in the symmetric limit because all chiral orientations are equivalent and related by symmetry. In the real world the explicit chiral symmetry breaking makes the solution of $\theta(\infty, \mathbf{r}) \neq 0$ unstable by the amount $\Delta V = f_\pi^2 m_\pi^2(1 - \cos\theta(\infty, \mathbf{r}))$ in energy density. By the time when the kinetic energy density of pion field decreases to be comparable to this energy density, our approximation of perfect chiral symmetry breaks down and we must start including an explicit chiral symmetry breaking. Let us make a quantitative estimate of this transition time.

To be concrete, we consider the simple DCC solution of Eq.(37) with $\theta_0(x)$ given by Eq.(28). The asymptotic $\theta(x)$ in the absence of symmetry breaking is therefore

$$\mathbf{n}\,\theta(x) \to \mathbf{n}_0\Big(\theta_L + \theta_R + \frac{a}{t^2 - r^2}\Big), \tag{40}$$

where

$$a = \int \bar{\sigma}(\theta', \phi')d\Omega'/4\pi, \tag{41}$$

according to Eq.(21). We can relate the magnitude of parameter a to the time when the symmetry breaking becomes nonnegligible, namely the transition time t_0 from the formation period to the decay period. Our chiral symmetric solutions do not make sense when the pion kinetic energy becomes comparable to the potential energy of symmetry breaking; $\dot{\boldsymbol{\pi}}^2/2 \sim \Delta V$. With our solution, this condition gives $2f_\pi^2 a^2/t^6 \sim m_\pi^2 f_\pi^2$ at locations away from the light cone. Therefore the symmetry breaking cannot be ignored after

$$t_0 = \left(\frac{|a|}{f_\pi}\right)^{1/3}. \tag{42}$$

The transition time is delayed near the light cone where the pion field is stronger than in the interior. Since the spherical source expands nearly in the speed of light, t_0 is equal to the radius of source R_0 at time t_0. After time t_0, the DCC starts decaying although the transition from formation to decay is not clear-cut. Meantime the source keeps expanding and weakening in strength. If R_0 is 5 *fm*, for instance, we obtain $|a| \sim 16 f_\pi^{-2}$. In order to generate a pion field of this magnitude and extent, we need, according to Eqs.(12), (19), (20), and (41), the integrated source strength of

$$\left|\int i(\overline{Q}\boldsymbol{\tau}\gamma_5 Q)d^3\mathbf{r}\right| = \frac{2\pi f_\pi^2 a}{m_Q R_0} \sim 13. \tag{43}$$

The axial isospin charge density in the DCC can be computed with Eq.(40). It decreases with time at a fixed location and increases at a fixed time as we approach the source. At the origin it reaches a value independent of the source strength parameter a by time t_0; $|\mathbf{A}_0(t_0, \mathbf{0})| = 2f_\pi^{-3}$. We do not discuss how easily a source of this magnitude can be produced in hadron-hadron collisions. We rather proceed to construct a continuous picture from formation to decay in the case of the simplest isospin-uniform DCC.

5 Connecting formation to decay

In this Section we smoothly connect the simplest chiral symmetric solution of Eq.(40) at $t < t_0$ to a solution at $t > t_0$ which attenuates with an explicit symmetry breaking. This boost-invariant solution is not only physically interesting but also easy to work with. Actually, as far as we have explored, this is the only

workable case where complete analytic study is possible. We add the symmetry breaking

$$\mathcal{L}_{br} = m_\pi^2 f_\pi^2 (\cos\theta - 1) \tag{44}$$

to the chiral symmetric Lagrangian of Eq.(1) after time t_0, turning the equation of motion for $\theta(x)$ into the sine-Gordon equation of $3+1$ dimensions. For the purpose of our analytic study, however, we reduce the equation of motion to a linear form by expanding $\cos\theta$ in $\mathcal{L}_{br}$ around $\theta = 0$ and keeping only the leading term. This approximation breaks down near the light cone, where the phase pion field θ grows beyond $O(1)$, but it is good enough everywhere else for most purposes. Then the equation for θ is simply the equation of free motion with mass m_π. Since the chiral symmetric solution in the formation period is boost invariant, we look for a boost-invariant decay solution to connect to it. Inside the light cone where there is no quark matter, the θ field obeys the differential equation,

$$\frac{1}{\tau^3}\frac{d}{d\tau}\left(\tau^3 \frac{d}{d\tau}\theta\right) + m_\pi^2 \theta = 0 \qquad (\tau^2 > 0). \tag{45}$$

The general solution is given by the cylindrical functions:

$$\theta(x) = \frac{c_1 J_1(m_\pi \tau) + c_2 N_1(m_\pi \tau)}{m_\pi \tau}, \tag{46}$$

where $J_1(z)$ and $N_1(z)$ are the Bessel and Neumann functions of the first order with c_1 and c_2 being constants. In the limit of $m_\pi \to 0$ (*i.e.*, $z \to 0$), $J_1(z)/z$ approaches $1/2$ while $N_1(z)/z$ behaves as

$$\frac{N_1(z)}{z} \sim -\frac{1}{\pi}\left(\frac{2}{z^2} - \ln\left(\frac{z}{2}\right) + \cdots\right), \tag{47}$$

We want the solution of Eq.(46) to turn into the chiral symmetric solution of Eq.(37) when $m_\pi \to 0$. This is realized if we choose the constants as

$$\begin{aligned} c_1 &= 2(\theta_L - \theta_R) \equiv 2\theta_\infty, \\ c_2 &= -\frac{1}{2}\pi m_\pi^2 a. \end{aligned} \tag{48}$$

At locations far away from the light cone, we find with the asymptotic formulas of the Bessel and Neumann functions that the pion field oscillates as

$$\theta(x) \sim \sqrt{\frac{16\theta_\infty^2 + \pi^2 m_\pi^4 a^2}{2\pi (m_\pi \tau)^3}} \cos\left(m_\pi \tau - \frac{3\pi}{4} + \vartheta_0\right), \qquad (\tau \to \infty) \tag{49}$$

where

$$\tan\vartheta_0 = \frac{\pi m_\pi^2 a}{4\theta_\infty}. \tag{50}$$

Attenuation of $\theta(x)$ is interpreted as decay of the DCC due to emission of pions with nonvanishing mass. The decay is not exponential, but obeys a power law $\sim t^{-3/2}$ in amplitude with the time scale of m_π^{-1}. The momentum spectrum of pions can be computed with the standard method [9].

Now that we have obtained a complete solution Eq.(46) with Eq.(48) from formation through decay including a symmetry breaking, it is appropriate here to make some quantitative discussion with our solution. Let us use for this purpose the set of parameters used earlier in Sec.IV; $t_0 = 5fm/c$ and $|a| = 16f_\pi^{-2}$. For these values, the tail of the spacetime dependent term $a/(t^2 - r^2)$ in the symmetric solution at the transition time t_0 is comparable or larger than asymptotic limit θ_∞ ($|\theta_\infty| < \pi/2$). As we go further down in time, the a-dependent part dominates over the θ_∞-dependent part by an order of magnitude in the asymptotic oscillation of Eq.(49). It means that distinction between the solution with $\theta_\infty = 0$ and those with $\theta_\infty \neq 0$ is insignificant even during the decay period. The origin of this rather unexpected result is traced back to the fact that the chiral symmetry breaking enters through m_π, which is numerically about the same in magnitude as the chiral symmetric energy scale f_π. If we set $m_\pi/f_\pi \to 0$ contrary to reality, the transition time t_0 would be stretched out as $t_0 \sim (f_\pi/m_\pi)^{1/3}(af_\pi^2)^{1/3}f_\pi^{-1} \to \infty$. Then the tail of the a-dependent term of the pion field $(a/{t_0}^2)$ would attenuate sufficiently by the time t_0 ($\theta(t_0) \sim (m_\pi/f_\pi)^{2/3}(f_\pi^2 a)^{1/3}$) so that the θ_∞ term would dominate over the a term both at t_0 and in the asymptotic oscillation. This does not happen at least in our solution. As for dependence on the strength of source $\rho(x)$, we can make the following statement: Since a is proportional to the strength of source and t_0 depends on a as $\sim a^{1/3}$, the transition is delayed for a stronger source, but not very sensitive to the source strength. Since the tail of the a-dependent term of $\theta(x)$ behaves like $\sim a^{1/3}$ at t_0, differentiating the DCC solution ($\theta_\infty \neq 0$) from the correctly oriented solution ($\theta_\infty = 0$) is a little harder when a DCC is generated by a stronger source.

The asymptotic $\theta(x)$ field of Eq.(49) is meaningless when the pion field becomes too weak. This terminal time t_f, which can be interpreted as the decay

lifetime of DCC, is set by the condition $|\theta(t_f, \mathbf{r})| \ll 1$. According to Eq.(49), this happens at

$$t_f \gg (2\pi f_\pi^4 a^2)^{\frac{1}{3}} m_\pi^{-1}. \tag{51}$$

The right-hand side is $\sim 17 fm/c$ for $|a| = 16 f_\pi^{-2}$. The decay proceeds very slowly in the time scale of hadron physics.

In this Section, on the basis of the simplest isospin-uniform DCC, we have attempted to build a semiquantitative picture of formation and decay of DCC. A missing information is how hadron-hadron collisions or heavy-ion collisions can possibly produce a strong enough isovector pseudoscalar density of quark. We will develop some qualitative argument in the final Section.

6 Discussion

It is interesting to note that the equation of motion for $\theta(x)$ in Eq.(8) is the statement of local conservation of the axial isospin current:

$$\mathbf{n}(x) \cdot \partial^\mu \mathbf{A}_\mu(x) = 0. \tag{52}$$

When it is put into the form

$$\begin{aligned} \mathbf{n} \cdot \partial^\mu \mathbf{A}_\mu^{(\pi\sigma)} &= -\,\mathbf{n} \cdot \partial^\mu \mathbf{A}_\mu^{(q)} \\ &= -\,i m_Q \mathbf{n} \cdot (\overline{Q} \boldsymbol{\tau} \gamma_5 Q), \end{aligned} \tag{53}$$

it is the equation of axial isovector charge transfer from the *current* quark to the $\boldsymbol{\pi} - \sigma$ system. The isovector pseudoscalar density of the *constituent* quark is the rate at which the transfer is made.

The axial isospin charge $\mathbf{A}_0 = Q^\dagger(\boldsymbol{\tau}/2)\gamma_5 Q$ takes a simple form in the infinite momentum limit. Let us examine for instance the third component $A_0^{(3)}$. In the valence approximation, the axial isospin charge for the nucleon is simply related to $g_A = 5/3$ of the famous SU(6) prediction or the constituent quark model prediction for the nuclear β-decay:

$$\begin{aligned} &\lim_{\mathbf{p}\to\infty} \langle \text{proton}(\mathbf{p}, h = \pm\frac{1}{2}) | A_0^{(3)} | \text{proton}(\mathbf{p}, h = \pm\frac{1}{2}) \rangle \\ &= \lim_{\mathbf{p}\to\infty} \langle \text{proton}(\mathbf{p}, h = \pm\frac{1}{2}) | \frac{1}{|\mathbf{p}|} \mathbf{p} \cdot \mathbf{A}^{(3)} | \text{proton}(\mathbf{p}, h = \pm\frac{1}{2}) \rangle \end{aligned}$$

$$
\begin{aligned}
&= \pm\frac{1}{2}\times\frac{5}{3}, \\
&\lim_{\mathbf{p}\to\infty}\langle \mathrm{neutron}(\mathbf{p}, h=\pm\frac{1}{2})|A_0^{(3)}|\mathrm{neutron}(\mathbf{p}, h=\pm\frac{1}{2})\rangle \\
&= -\lim_{\mathbf{p}\to\infty}\langle \mathrm{proton}(\mathbf{p}, h=\pm\frac{1}{2})|A_0^{(3)}|\mathrm{proton}(\mathbf{p}, h=\pm\frac{1}{2})\rangle. \qquad (54)
\end{aligned}
$$

For the quarks, $A_0^{(3)}$ takes opposite signs for helicity $+1/2$ and $-1/2$ states and also for the u-quark and the d-quark. The sign remains the same for a quark and an antiquark since $\mathbf{A}_\mu$ is even under charge conjugation.

The axial isospin charge is small in the initial state of the $\bar{p}p$ collision. Unless chiral symmetry is badly broken in hard collisions, the total axial isospin charge of the quark-pion system should remain close to this small number after collision. In order to produce a large axial isospin in the quark source on the shell, therefore, the small axial charge must be polarized into a pair of large charges of opposite signs, one in the quark-antiquark source within the shell and the other in the DCC pions. Transfer of the axial charge between them acts as the source term to create a DCC. Our postulate on the time dependence of the quark source in Sec.III is consistent with this observation: Increasing isovector pseudoscalar quark charge would mean that axial isospin charge transfer accelerates with time to polarize even further without limit. A natural scenario is that a strong polarization of axial isospin charge is created initially and gradually depolarizes as the source feeds the DCC pion field in the interior during the formation period.

However, in order for a constituent quark source to acquire a large axial isospin, the u-quark and the d-quark must react differently in the initial hard collision: According to Eq.(54), the helicity $+1/2$ state dominates over the $-1/2$ state for the u-quark while the $-1/2$ state dominates over the $+1/2$ state for the d-quark. How can such a flavor dependence arise from the fundamental dynamics that is flavor blind ? A flavor dependence generated by a statistically random process is far too small. An answer to this question must come from quantum chromodynamics in high density and at high temperature. If creation of a large isovector pseudoscalar density of quark is impossible, formation of a spectacular Centauro or an anti-Centauro would be ruled out. However, this does not prohibit formation of nonuniform DCCs which are related to them

through chiral rotations. This may be the reason why the classical pion field seen in the numerical simulation [3] exhibits a textured isospin orientation, not a uniform alignment of isospin.

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